Mathematical Model for Estimating Flood Disaster Effect on a Population by using Differential Equation

Shubham Kumar[*1] *, Rajan Singh*[2] *, Dr. B.K.Singh*[3]*, Riddhi Garg*[4]

[1] Research Scholar, Department of Mathematics, IFTM University, Mooradabad 244001, India
[2] Assistant Professor, Department of Mathematics, IFTM University, Mooradabad 244001, India
[3] Professor, Department of Mathematics, IFTM University, Mooradabad 244001, India
[4] Assistant Professor, Department of Mathematics, IFTM University, Mooradabad 244001, India

Abstract: *Disaster management is very much needful topic for discussion. Disasters are mainly of two kinds manmade or natural, but their impact on human life are very much lethal. Natural disasters are earthquake; floods, hailstorm etc. and manmade disaster are pollution, war, terrorism, fires, power failure etc. The main objective of this study is to prepare a suitable mathematical model for estimating the effect of flood disaster on a population. India is one of amongst country which is suffering from many kind of natural and manmade disaster from time to time. Many state of India specially Uttrakhand, Bihar, Bengal etc. are facing natural disaster problems like earthquakes, sea storms, floods, landslide, hail storm, snow avalanches etc. it is very much essential that a proper system should exist in order to decrease the loss of human life as well as infrastructure.*

Keywords- Population, Flood disaster, prone, Population size, Limits, Integration, Convergence, India.

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1. Introduction

One of the most common natural disasters, but also one of the most lethal, is flood. These take place in many different countries all over the world, particularly during the heavy rain season, and can be caused by a range of different things. Some of the things that can start the floods can be totally natural, while others can be manmade, but the speed at which they spread is entirely down to nature. The flood disaster can take place in both plane and hill regions. The two natural causes of floods are the heavy rain, while they can also be caused by water dam braking, soil erosion etc. Flood propagation modeling can be defined as the art of quantitatively describing the characteristics and evolution of the flow that is set up when a large amount of water moves along the earth surface in an uncontrolled way.

Natural disasters and climate change are fast emerging as the most defining challenges of the 21st century. India's unique geo-climatic condition makes it highly susceptible to climate change and natural disasters like floods. The country has observed significant anomaly in natural variability of temperature and rainfall patterns and has experienced more frequent and lethal disasters in recent decades. In this paper we will define some important variables to estimate the fact, how a flood disaster spread in a region and how it affects the population in a certain region.

2. Research Methodology-

There are several methods for estimating effect of a disaster, hits a specific region, but they do not necessarily give same results. The method we opted in this mathematical model is not a single unique method but a set of techniques. This method consists of differential equation as well as theory of limits and convergence. It is an important method for testing the effect of one variable on others.

The following research methodologies are adopted for the proposed research paper:

- Identification of the problem and defining the variable for study
- Collection and study of available related literature
- Mathematical formulation of the problem by using differential equations.
- Mathematical solution of the model
- Interpretation of results.
- Conclusion

3. A simple deterministic Model

In a given population at a time t, let *S(t)* be the number of flood disaster affected susceptible persons for, i. e., the number of persons those who can be affected , *I(t)* be the number of flood disaster affected person in the population, and *R(t)* be the number of those removed from the population recovery, lost , death, hospitalization or by any other means. If *N(t)* is the total population size, we have

$$S(t) + I(t) + R(t) = N(t) = \text{Constant} \qquad (1)$$

We first consider simple deterministic models, i.e., models in which there are no removals. Let n be the initial number of flood disaster affected susceptible in the population in which one flood disaster affected person has been introduced so that

$$S(t) + I(t) = n + 1, \quad S(0) = S_0 = n, \quad I(0) = I_0 = 1 \qquad (2)$$

Now due to flood disaster, the number of flood disaster susceptible persons decreases and the number of affected persons increases. We assume that the rate of decrease of *S(t)*, or the rate of increase of *I(t)*, is proportional to the product of the number of susceptible and the number of affected so that our model gives

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI \qquad (3)$$

Where, β is constant of proportionality

From equation (2) and (3)

$$\frac{dS}{dt} = -\beta S\ (n + 1 - S\ ) \qquad (4)$$

Solving equation (4), we get

$$\frac{dS}{-S\,(n+1-S)} = \beta dt$$

$$\frac{dS}{S\,\{S-(n+1)\}} = \beta dt \qquad (5)$$

Dissolving $\frac{1}{S\,\{S-(n+1)\}}$ into partial fraction

$$\frac{1}{S\,\{S-(n+1)\}} = \frac{A}{S} + \frac{B}{\{S-(n+1)\}} \qquad (a)$$

$$\frac{1}{S\,\{S-(n+1)\}} = \frac{A\{S-(n+1)\} + B\,S}{S\,\{S-(n+1)\}}$$

$$1 = A\{S-(n+1)\} + B\,S \quad (b)$$

Putting S= 0 in eqn. (b)

1 = A {-(n+1)} + 0

So, $A = \frac{-1}{(n+1)}$

Putting S= n+1 in eqn. (b)

1 = 0+ B (n+1)

So, $B = \frac{1}{(n+1)}$

So, from eqn. (a)

$$\frac{1}{S\,\{S-(n+1)\}} = \frac{1}{(n+1)}\left[\ \frac{1}{\{S-(n+1)\}} - \frac{1}{S}\ \right\}$$

So, from eqn. (5)

$$\beta dt = \frac{1}{(n+1)}\left[\ \frac{ds}{\{S-(n+1)\}} - \frac{ds}{S}\ \right]$$

On integrating both sides

$$\beta t = \frac{1}{(n+1)}[\ \log \{S- (n+1\} – \log s)] + C \qquad (c)$$

Where, C is a constant.

Since $S\,(0) = S_0 = n$

So from eqn. (c)

$$0 = \frac{1}{(n+1)}[\ \log \{\ n- (n+1\} – \log n)] + C$$

$$0 = \frac{1}{(n+1)}[\ \log (-1/n)] + C$$

$$C = -[\ \log (-1/n)]$$

So from eqn. (c)

$$\beta t = \frac{1}{(n+1)}[\ \log \{S- (n+1\} – \log s)]\ -[\ \log (-1/n)]$$

$$\beta t = \frac{1}{(n+1)}\log[\ \frac{\{S- (n+1\}}{S}]\ -[\ \log (-1/n)]$$

$$\beta t = -\frac{n}{(n+1)}\log[\ \frac{\{S- (n+1\}}{S}]$$

$$(n+1)\,\beta t = -n\log[\ \frac{\{S- (n+1\}}{S}]$$

$$e^{(n+1)\,\beta t} = -\,n[\ \frac{\{S- (n+1\}}{S}]$$

$$e^{(n+1)\,\beta t} = -\,n[\;1 - \frac{(n+1)}{s}\;]$$

$$e^{(n+1)\,\beta t} = -\,n + \frac{n(n+1)}{s}$$

$$\frac{n(n+1)}{s} = n + e^{(n+1)\,\beta t}$$

$$S\,(t) = \frac{n(n+1)}{n + e^{(n+1)\beta t}}\;,$$

Since $S\,(t) + I(t) = n + 1$

So, $I\,(t) = (n + 1) - S(t)$

$$I\,(t) = (n + 1) - \frac{n(n+1)}{n + e^{(n+1)\beta t}}$$

$$I(t) = \frac{n(n+1)e^{(n+1)\beta t}}{n + e^{(n+1)\beta t}}$$

3. Result

$$\lim_{t\to\infty} S(t) = 0;$$
$$\lim_{t\to\infty} I(t) = n + 1$$

And ultimately, all persons of a certain region will be affected by the flood disaster

5. Conclusion-

The main merit of this method is that it is relatively simple, can be easily understood and involve simpler calculations. Here we have defined the variables *S(t), I(t) , R(t)* as a function of a variable t. With the help of this model we can say that flood disaster is one of the most lethal kinds of disaster. This model shows that when a flood disaster occurs at a place it is very difficult to control it. Flood disaster affected the persons of a certain place in a huge number.

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